

$$S_x = \frac{P_1[2ab + (a^2 - b^2) \cos 2\alpha - (a + b)^2 \cos 2(\alpha - u)]}{a^2 + b^2 - (a^2 - b^2) \cos 2u} \quad (30)$$

tensile stresses are negative.

The stress at the crack tip is given by equation 30 with $u = 0$. For a flat crack, a is much greater than b . Then, equation 30 can be written as equation 31,

$$S_x = (P_1/b)(a - (b + a) \cos 2\alpha) \quad (31)$$

Equation 30 shows that if P_1 is tensile, the stress at the crack tip is tensile except where α is very close to zero; that is, when the major axis of the crack is nearly parallel to the applied tension. If P_1 is compressive, S_x is compressive except when α is very close to zero. The maximum value of the tensile stress at the crack tip is $P_1(1 + 2a/b)$ when P_1 is tensile ($\alpha = 90^\circ$) and P_1 when P_1 is compressive ($\alpha = 0$).

When P_1 is compressive and $\alpha = 0$, notice that the tensile stress at the crack tip is independent of the form of the crack. Unless P_1 approaches the tensile strength of the atomic bonds at the crack tip, the crack cannot propagate catastrophically.

Hoek [1965, p. 16] pointed out that, while the maximum tensile stress tangential to the crack surface of flat cracks occurred near the crack tip, it did not occur at the crack tip. He simplified equation 30 by assuming that u is small, and b is small compared to a [Hoek, 1965, appendix 1]. By differentiating the resulting expression with respect to u , Hoek was able to show that the maximum tensile stress S_t near the crack tip is given by

$$S_t z_0 = P_1(\sin^2 \alpha \pm \sin \alpha) \quad z_0 = b/2a \quad (32)$$

When P_1 is compressive, the negative sign in equation 32 is appropriate; S_t will always be negative (tensile) when P_1 is compressive, except when $\sin \alpha = 1$ or 0, then S_t is indeterminate. Notice that, as the positive sign in equation 32 should be used when P_1 is tensile, S_t is always tensile and considerably larger than its value when P_1 is compressive.

Hoek's approximation leads to errors when α is close to zero or 90° . This can be seen by comparing equation 32 with equation 31, (which is exact) or from the predicted positions of S_t .

These are given by equation 33 [Hoek, 1965, appendix 1].

$$V_t = -b/2a(\tan \alpha \pm \sec \alpha) \quad (33)$$

The errors arise because some products of b and trigonometric functions of α removed by the simplification of equation 30 are not negligible when the trigonometric functions take extreme values.

A more elaborate analysis than Hoek's is required to determine the exact situation. It will not be attempted here. Instead, notice that symmetry considerations suggest that the maximum tensile stress is at the crack tip when the crack major axis is parallel or perpendicular to the principal stress, and that equation 33 suggests that, in other positions, the maximum tensile stress is at some distance from the crack tip.

The situation is more complex when the crack is closed. Hoek [1965, p. 24] used the same approximations as he made in the case of open cracks to show that on McClintock and Walsh's hypothesis of the behavior of closed cracks,

$$S_t z_0 = P_1 \sin(\cos \alpha - m \sin \alpha) \quad (34)$$

where m is the coefficient of friction on the crack surface. The stress S_t is tensile for values of $\cos \alpha$ greater than $m \sin \alpha$. Taking m to be equal to one, closed cracks inclined at more than 45° to P_1 will not, then, grow in uniaxial compression.

A NEW THEORY OF BRITTLE CREEP

We now use this discussion of stress distribution around cracks and Charles's theory to explain brittle creep in uniaxial compression.

Suppose that a subcritical crack in uniaxial compression extends in its own plane by stress corrosion due to the tensile stress near the crack tips, and that when it reaches a critical length, it propagates in the manner described by Brace and Bombolakis [1963].

This sequence may seem less plausible than assuming that the crack grows along the path of a hypothetical branch fracture. However, the alternative leads the crack to a stable configura-